Airplane Engine Selection by Optimization on Surface Fit Approximations

Michael J. Healy*
Boeing Computer Services, Inc., Seattle, Wash.
and
Janusz S. Kowalik†
Washington State University, Pullman, Wash.
and
James W. Ramsay‡
Boeing Commercial Airplane Company, Seattle, Wash.

A comprehensive engine/airframe screening methodology has been developed based on surface fitting and nonlinear optimization procedures. These procedures include the use of experimental design techniques, and a gradient-based method for nonlinear constrained minimization. The methodology has been programed for use on the CDC 6600 computer and has been successfully demonstrated on extensive test cases. One test case involved selection of an optimum airplane design using performance data for only 28 designs. A total of 256 designs were required to locate this optimum graphically.

Introduction

THE methods described in this paper were originally developed for economically screening a variety of proposed aircraft engine concepts using airplane performance predicted by computer simulation. Evaluation of these engine concepts requires simultaneous estimation of the effects of several independent engine and airframe design variables. These include engine parameters such as cycle pressure ratio (CPR), turbine temperature (T4), bypass ratio (BPR), engine size (SCA), and airframe parameters such as wing size (W/S), aspect ratio (AR), thickness-to-cord ratio (t/c), and airplane thrust-to-weight ratio (T/W). The effects of these variables are expressed as values of a set of aircraft design performance indicators, or response functions, such as range (RANGE), takeoff gross weight (TOGW), takeoff field length (TOFL), thrust margin (TMDOD), and specific excess power (PSUBS). Numerical values for these response functions are output by the computer simulation for specified input values of the independent design variables.

With the advent of computer models of installed-engine performance in aircraft, advance screening of proposed designs has become possible. However, the simulations involved in a proper parametric evaluation of installed-engine performance are quite complex. For example, the Boeing Engine/Airframe Matching Program (BEAM) does performance analysis by simulating one or more missions flown by a mathematically defined airplane which has been sized on a baseline mission in BEAM. The airplane is defined in the program input by specifying values for the design variables and various program settings which describe airframe configuration, engine placement, etc. In fact, the simulation is a two-stage process (Fig. 1), requiring a separate computer program to compute performance for the engine design, which is then input to the BEAM program to obtain airplane system performance for any desired combination of airframe design values.

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*Mathematical Analyst, Mathematics and Modeling Unit.

†Associate Professor, Department of Mathematics and Computer Science.

‡Specialist Engineer, Noise Research Unit.

A comprehensive parametric study of airplane designs requires the numerical evaluation of the many performance response functions at potentially thousands of combinations of values of the independent design variables. For example, 6 variables with 3 allowed values, or levels, per variable will yield 729 total combinations. To properly measure the nonlinear effects of the variables individually and in combination requires the consideration of at least 3 levels per variable, although it might be possible to eliminate many of the combinations.

Given the complexities of calculation and the volume of data involved, it is evident that if the effects of the design variables are to be estimated by interpolating directly on reponse function data, gross simplifications must be made to avoid large expenditures in engineering and computer time. For example, the number of variables must be kept small, which causes significant effects of the variables and their interactions to be left out of the analysis. It will be shown that computer resources can be drastically cut and engineering resources augmented by the application of automated techniques of screening using limited quantities of simulation data.

To provide a basis for the screening analysis, the screening process is formulated in terms of an optimal design problem. To find design variable values yielding superior performance,

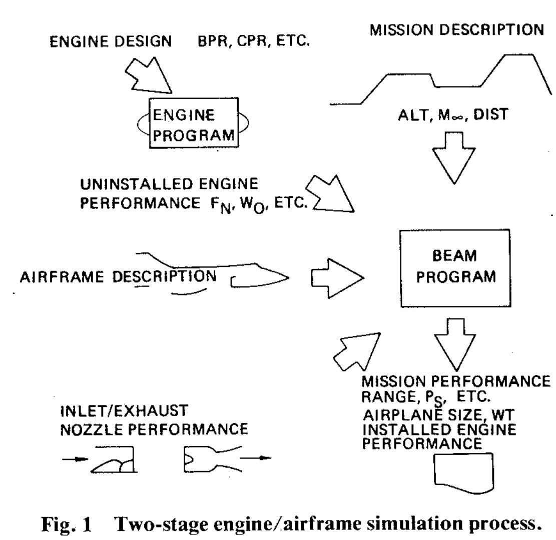


Fig. 1 Two-stage engine/airframe simulation process.

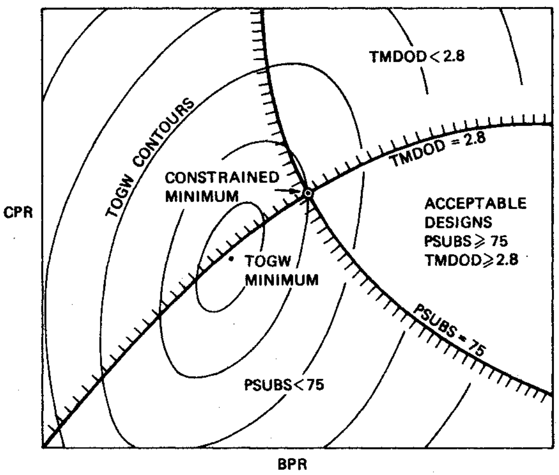


Fig. 2 Hypothetical optimization problem.

a cost response (such as TOGW) is minimized subject to performance level constraints on other response functions. It will be shown that available techniques for nonlinear constrained optimization can be used to locate optimal combinations of engine and airframe design variable values based on response function values for a number of nonoptimal designs. The effects of the variables on airplane system performance can then be investigated in a highly restricted neighborhood of designs which are "close to" the calculated optimum. If the optimization can be performed using a small enough fraction of the engine program and BEAM simulator runs which were projected for a complete analysis, a detailed parametric screening analysis can be realized.

Problem Definition

The airplane response functions are functions which vary in some dependence relationship with the independent design variables which are input to the computer simulation. Obtaining explicit mathematical expressions for the dependence relationship involved is impractical because of the complicated logic flow of the simulator. This eleminates the possibility of bypassing the highly complex simulation process with an explicit analytical model which gives function values in an economical fashion.

Response function values are needed, however, at a number (which could be quite large) of nonoptimal design points to solve an optimal-design problem by available methods. To describe such an optimal-design problem is the purpose of this section. An example (unrealistically small, for clarity) will be given. The next section will then show how the problem of obtaining response function values economically is solved.

One way to specify which airplane designs are optimal is to select a performance indicator such as TOGW which is regarded as a cost response, one whose value should be as close as possible to a minimum. Alternatively, a response function may be maximized by converting it to a cost response, for example, but minimizing the negative of the function. Hence, optimization will be treated here as minimization of a cost response function subject to performance level constraints on the remaining response functions.

An example of an optimal-design problem stated according to these criteria is to minimize TOGW subject to constraints on the minimum values of the response functions PSUBS and TMDOD. These contraints are inequalities; e.g., PSUBS $\geq$ 75 and TMDOD $\geq$ 2.8. This problem is illustrated in Fig. 2 by showing contours of TOGW, PSUBS, and TMDOD as functions of the two variables BPR and CPR. Forcing TOGW close to its minimum subject to the constraints results in optimal design values for BPR and CPR. The optimum is shown as a point in design space at the intersection of the PSUBS and TMDOD contours which just meet the performance constraints.

In addition to the nonlinear constraints on BPR and CPR values which are given by the inequalities for PSUBS and TMDOD, the independent design variables are constrained directly by selecting a boxlike *region-of-interest,* within which an optimal design is supposed to lie. This is done to limit the data-handling problem for the airplane simulations on the computer and is basic to the methodology used here for the screening process. The region-of-interest constraints are simple lower and upper bound specifications for BPR and CPR; for example,

$$0.2 \leq \text{BPR} \leq 2.0 \tag{1a}$$

$$12.0 \leq \text{CPR} \leq 30.0 \tag{1b}$$

Optimization with Approximating Functions

There exist nonlinear optimization algorithms which search in an iterative fashion for an optimum through a sequence of nonoptimal design points. Starting values for the design variables are input to the algorithm and the search proceeds with the algorithm calling for response function values (and perhaps partial derivatives with respect to the design variables) to determine the next search point in design space. After iterating through a number of nonoptimal designs, an optimum is reached if it exists. Since response function values are required at each iterative search point, the problem of limiting the number of airplane simulations must be faced, even when an optimization search is used as described.

It is not feasible to let the optimization algorithm call directly for response values from the simulator; hence, approximating functions are used to give approximate response values whenever needed. These approximating functions are simple mathematical expressions, such as polynomials in several variables. The coefficients for the terms involved can be found by analyzing true response values at a number of design points; for example, by the method of least squares, in such a way that approximation error is kept to a minimum. It will be shown that good approximations can be obtained in this way using a relatively minor amount of actual simulation data. The success of other investigators using approximating functions in a similar optimal-design problem is described in Refs. 1 and 2.

In addition to limiting the number of simulations, approximating functions (because of their simple analytical form) make possible the economical calculation of partial derivatives of each response function with respect to each design variable; e.g., $(\partial \text{TOGW})/(\partial \text{BPR})$. This opens the way for the use of gradient-directed optimization techniques, which utilize the information in the gradient vector of partial derivatives in addition to function values to locate the minimum of a function of several variables. Gradient techniques are generally much more efficient[3] than other minimization techniques. Also, partial derivatives are of use in the final screening analysis in a close neighborhood of a calculated optimum.

Obtaining the Approximating Functions

The problem at hand is a typical design problem where function evaluations are very expensive and the direct application of an optimization method is not possible. There are 2 ways of handling the difficulty: a) *model simplification,* where the complexity of mathematical model corresponding to the design problem is reduced and a simplified formulation is derived: b) *simple functions* are used to approximate the true response functions defining the design problem. This can be done either by a global fit over the entire area of search or by a piece-wise fit, if necessary.

The first approach has serious limitations, since beyond certain simplifications the model becomes inaccurate and unrealistic. In the case of the airplane design simulation, the complexities of the model prohibit the derivation of mathematical formulas for the response functions by problem analysis. Hence, the second approach has been selected, bypassing an intensive analysis of the simulator functions by using statistically-controlled approximations derived from limited data.

For simplicity and reliability, low-order polynomials in n variables are used for response function approximations. The coefficients for the polynomial terms are calculated in a regression program which provides goodness-of-fit statistics in addition to coefficients, based on actual response function values corresponding to design variable values. A least-squares surface fit in two design variables, BPR and CPR, using a quadratic polynomial to approximate the true response surface, is illustrated in Fig. 3. The nonzero coefficients for the polynomial are determined in a stepwise regression analysis on the data. A term which is deemed not significant is assigned a zero coefficient. In this way, those design variables and their interactions which appear significant according to the test criteria are isolated, and this aids in selecting important design effects for further analysis.

For clarity, an outline of the revised engine screening process is given before going into more detail about the methods used: a) Response function data is gathered at carefully selected values of the design variables. b) Surface fits are obtained by regression on the date. c) The surface fits are statistically tested for validity. d) One or more optimal designs are located by an optimization program which interrogates the surface fit polynomials to provide function and gradient vector values. e) An intensive parametric analysis is done for designs lying in a close neighborhood about selected local optima.

The fifth step is carried out not only by running the simulation on selected designs near the local optima, but by doing a comprehensive sensitivity analysis using the surface fit polynomials. Surface fit function and partial derivative evaluations can be done many times very economically to identify important effects near an optimum. Also, if the optimization program is highly efficient, the problem can be redefined by placing a tolerance on cost increase going away from the optimum and reoptimizing a number of times to find acceptable cost-increase limits on the range of each design variable. For example, if $TOGW_{opt}$ is the optimal cost and TOL a tolerance value on nonoptimal cost, the new constraint

$$TOGW \leq TOGW_{opt} + TOL$$

can be added to the other inequality constraints and minimization or maximization of each design coordinate carried out in a series of reoptimizations to find cost-increase limits on the design variables, centered about the optimum.

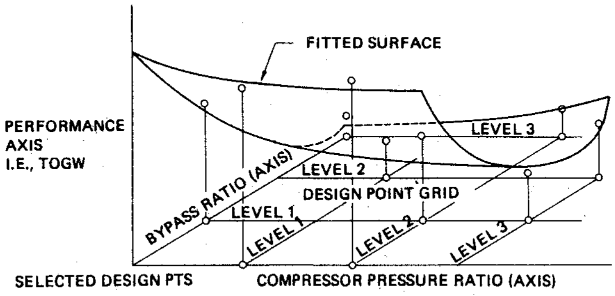
Experimental Design

Selecting the design combinations for the regression surface fit data must be done with some care and involves the field of experimental design.[4] Any data bias in favor of certain design variables or the buildup of significant cross-correlations between the supposedly independent design variables will adversely affect the surface fit coefficients. Important trends in each response function must be correctly translated by the approximating functions. Methods used for the selection and analysis of experimental data are helpful at this stage.

Selection of the type of approximating function is also a part of experimental design, and it has already been indicated that multivariable polynomials in the n design variables are being used. In fact, the simplest nonlinear polynomial, a quadratic, has been used exclusively for the screening methodology described here at this writing.

Each response function is a function $y=f(x)$ of an n-dimensional vector x of design variables $x_1, x_2, \ldots, x_n$. The particular selection of design variables x_i for a screening analysis dependes on engineering judgment and is problem-dependent. For example, one might consider

$$TOGW = f(BPR, CPR, T_4, SCA, W/S, AR, t/c)$$



$$TOGW = C_0 + C_1\,(BPR) + C_2\,(CPR) + C_3\,(BPR)^2 + C_4(CPR)^2 + C_6(BPR)\,(CPR)$$

Fig. 3 Least-squares surface fit.

where in this case $n=7$. Response values $y_k = f(x^{(k)})$ are obtained as simulator output at selected design combinations $x_1^{(k)} = (x_1^{(k)}, \ldots, x_n^{(k)})$, where $k=1,\ldots, K$, K being the total number of designs selected at which to gather data for surface fitting the f function. The number K must exceed the number of terms in the approximating polynomial, so that an over-determined system of linear equations

$$b_0 + \sum_{i=\Sigma}^{n} b_i x_i^{(k)} + \sum_{i \leq j} b_{ij} x_i^{(k)} x_j^{(k)} = y_k,\ k=1, \ldots, K \qquad (2)$$

in the unknown coefficients b_0, b_i, and b_{ij} can be solved by the method of least squares.

The greater the number K of equations, the more representative of the true response function the resulting polynomial is likely to be. However, each Eq. (2) corresponds to an evaluation of the response y_k in a simulator run for design point $x^{(k)}$, and actual response evaluations were to be eliminated as much as possible. The number of terms in a quadratic in n variables is $N=(n+1)\ (n+2)/2$, which is slightly in excess of $\frac{1}{2}n^2$. To keep K larger than N while minimizing simulator runs, integer multiples of n^2 have been tried to find a good value for K.

The goodness-of-fit of the regression surface is tested statistically by variance analysis. The statistics used most of all are the standard F-statistic for regression[5] and the multiple correlation coefficient squared (MCC^2 or R^2). The F statistic is defined as the ratio of the regression mean square,

$$\sum_{k=1}^{K} (\hat{y}_k - \bar{y})^2 / N \qquad (3)$$

to the error mean square,

$$\sum_{k=1}^{K} (y_k - \hat{y}_k)^2 / (K-N) \qquad (4)$$

where $\bar{y}$ is the mean of true response values y_k in the K data points and $\hat{y}_k$ is the response predicted by the polynomial at data point k. For a good fit to the data, the F value should exceed the F value in the standard tables for N and $K-N$ degrees of freedom and 95% confidence.

The MCC^2 is the ratio of the sum-of-squares in the numerator of the quotient (3) to the data value sum-of-squares, which is obtained by replacing $\hat{y}_k$ by y_k in the numerator of Eq. (3). This measures the fraction of the total variation in the data which is "explained" by the regression polynomial, and hopefully this fraction will be close to 1.0, exceeding, say, 0.90. Detailed interpretation of the statistical methods used in conjunction with experimental design can be found in any of Refs. 4-6.

The distribution of data points in design space is critical to the experimental design method used. To eliminate data bias and isolate the effects of the different independent design variables, a data distribution based on orthogonal latin squares[7] is used for the methodology of this paper. The

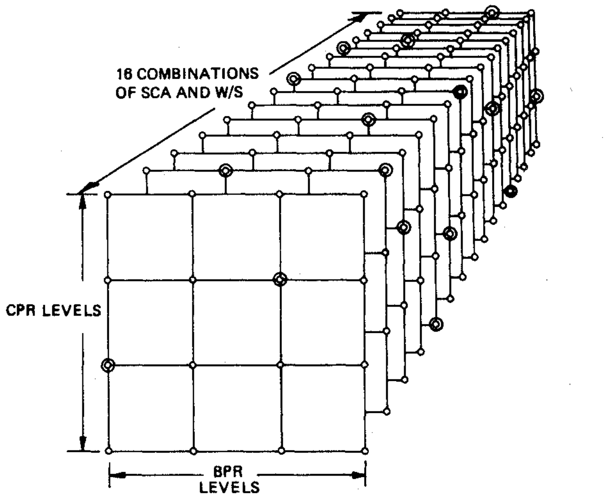


Fig. 4 Sparsity of selected data points in factorial design matrix for 4-dimensional case.

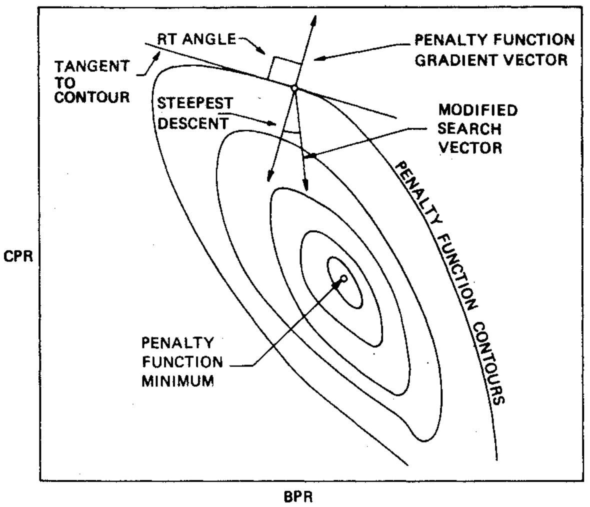


Fig. 5 Penalty function step *R*.

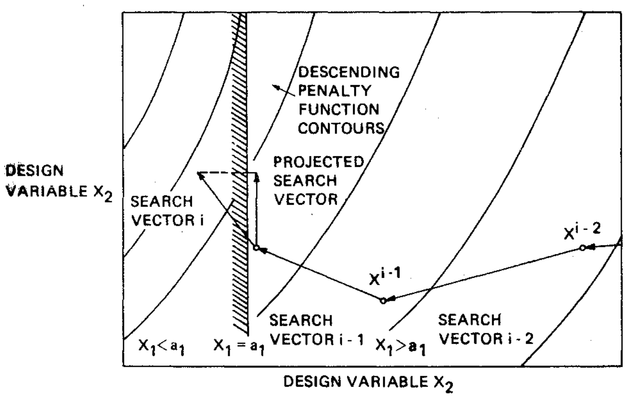


Fig. 6 Search vector projection.

distribution obtained has the property of orthogonality in that cross-correlations between the design variables are eliminated, and the data can be well-dispersed throughout a boxlike region-of-interest, such as Eq. (1). An idea of the relative sparsity of data from latin squares in comparison to the full factorial matrix of all possible combinations of design variable values is given in Fig. 4. In this example, there are four variables and four allowed values per variable. The full factorial matrix of 256 designs is well-distributed and orthogonal, but much smaller subsets having these properties can be obtained, as illustrated by the circled dots. Other experimental designs are discussed in Refs. 4 and 6.

Constrained Optimization via Penalty Function

To take full advantage of the properties of the simple approximating functions used in the revised screening methodology presented here, an efficient method for nonlinear constrained optimization is used. The method has two main features. These are a) an up-to-date gradient method for minimizing an unconstrained nonlinear function of several variables,[8] and b) a simple penalty function formulation, which transforms the constrained optimization problem into an unconstrained minimization problem. A comprehensive survey of optimization methods is contained in Ref. 3.

The penalty function/gradient algorithm used is a key part of the methodology of this paper. A number of different methods could be used for optimization. However, the selected method uses the approximating functions in a highly efficient manner and makes practical the repeated use of optimization. This may be needed to find all local optima which may exist, and it can aid in the intensive parametric analysis about each selected optimum. In particular, as an aid to the intensive final analysis, reoptimization with a tolerance on cost increase can be performed in the manner referred to earlier.

In applying the penalty function (Fig. 5), the problem of minimizing a cost response subject to constraints on other reponse functions is solved by minimizing several unconstrained functions sequentially. These unconstrained functions F_R, $R=1,2, \ldots$, are successive modifications of a basic penalty function. For example, if TOGW is the cost response, the penalty function F_R has the form

$$F_R(x) = \text{TOGW} + P_R \sum (\text{C.V.})^2 \tag{5}$$

Here, C.V. means "constraint violation," and the summation runs over all the constrained response functions which are in violation of their performance constraint values at design point x. For example, if PSUBS is constrained to the values PSUBS$\geq$ 75 and falls below the value 75, the term (PSUBS—75)^2 appears in the summation. The squared violations are all positive numbers and, hence, increase the value of $F_R(x)$, which is to be minimized.

The inequalities in the constrained problem are transformed out of the problem by Eq. (5). During the Rth minimization, the terms in the summation place a penalty on constraint violation, the severity of which is modulated by the weight factor $P_R>0$ (Fig. 5). Following minimization R, P_R is increased sharply to get P_{R+1} to intensify the effect of the constraints for minimization R+1. Let $x(P_R)$ be the calculated minimum of F_R. The sequence of designs $x(P_R)$ tends to the solution of the constrained problem [9-11] for carefully-selected increasing values of P_R.

The penalty function method will, if properly applied, locate the local optimum design nearest the starting point of the first minimization. The simple penalty function form shown here works quite well in the considered problem where the approximating functions are simple and an up-to-date gradient minimization method is used. Alternatively, more recent methods [12] resembling the penalty function formulation but having better numerical properties can be used.

The simple box constraints; e.g., in Eq. (1), are not incorporated in the penalty function. Because of their simple form, they lend themselves quite well to being resolved by a projection technique. This method projects the minimization search vector onto a direction parallel to all box constraints which the search threatens to cross during a minimization search (Fig. 6).

Test Result

In this section, 2 test cases for the screening methods in this paper are shown. The first test case involves 4 design variables and 5 airplane performance response functions, and is presented largely as a comparison of the effect of variations in data selection on the optima which are obtained using the suface fits obtained from the data. The second case, involving 6 design variables and 10 response functions, is a further test from a number of airframe optimizations with a fixed engine design. In each case, the response functions were approximated by quadratic polynomials in the design variables. The approximating function coefficients were obtained by regression analysis on very little data compared to that which would be needed for a complete analysis by cut-and-try methods. The surface fit approximating polynomials were tested using the F test and MCC^2 statistics described earlier, and the optimization program was applied to locate optimal designs. These were then compared with results obtained by actual simulator runs or by extrapolating from previously obtained simulator data.

In both cases, TOGW was the cost response and the remaining response functions were required to meet performance level constraints. The design variables were box-constrained during optimization, and box limits on each variable were set to the lower and upper values of that variable in the data selection. Surface fit data were obtained by assigning a number of equally spaced values throughout the range of each variable and selecting latin square subsets of the possible combinations of the variables at their respective allowed values.

The 4-variable test case was not a realistic case. It was intended as a test of the effect of a) the number of design points which should be used for data, b) whether orthogonal latin squares (OLS) were a significant improvement over a random scatter of well-dispersed design points in 4-dimensional space (the SLS), and c) which mode of running the regression program was best. The regression can be done in 3 different modes, forced fit, forward step, and backward step. Forced-fit mode is simply the elimination of stepwise analysis: all terms in the suface-fit polynomial have calculated coefficients whether or not they are significant in correlation with the response data which is being approximated. In forward-step mode, terms are entered in a stepwise fashion until all significant terms have been assigned coefficients. Backward-step mode is this process in reverse, except that the test criteria for keeping a nonzero term in the polynomial are less stringent and hence more terms usually appear in the resulting polynomial with nonzero coefficients, beginning with a forced fit as the first step.

In the case of 4 variables, $n^2 = 16$, and it was mentioned earlier that multiples of n^2 have been tried to find the minimum number of data points needed for a good surface fit. For a quadratic polynomial in this case, the maximum number of nonzero terms is $N = (n+1)(n+2)/2 = 15$. In actuality, 28 (approximately $2n^2$) and 52 (approximately $4n^2$) were the sizes of the orthogonal latin square (OLS) subsets which were tried, and there were 48 data points in the random (SLS) data selection. There were four allowed values in the range of each variable, hence, the factorial matrix of all possible combinations would have 256 data points in design space. In fact, all 256 designs were generated so that an optimum could be found by manual examination of computer graphics plots.

This is probably the largest number of mixed engine/airframe variables for which an optimum could be faired by hand from the data. In addition to the 256-point factorial matrix, nine additional designs were generated so that the central composite design,[4,6] a type of data selection which is not compared here, could be tried.

The variables used were bypass ratio (BPR), cycle pressure ratio (CPR), engine size (SCA), and wing size (W/S). BPR and CPR are engine variables and SCA and W/S are airframe variables in this analysis.

The 5 response functions for this test case were TOGW (the cost response), thrust margin (TMDOD), specific excess power (PSUBS), thrust-to-weight ratio (T/W) obtained for this case as a response instead of an independent design variable, and take-off field length (TOFL). The optimization problem was formulated as follows:

minimize	TOGW			
subject to	TMDOD		≥	2.80
	PSUBS		≥	75.00
	T/W		≥	0.60
	TOFL		≤	5000.00 ft
with	0.20	≤ BPR	≥	2.00
	12.00	≤ CPR	≤	30.00
	1.85	≤ SCA	≤	2.60
	170.00	≤ W/S	≤	215.00

The first 4 inequalities are nonlinear constraints, hence, they were incorporated in the penalty function. The simple box constraints on the design variables were applied by projection (Fig. 6).

Table 1 shows typical regression F test and MCC^2 statistics.

Table 1 Regression statistics—28-point orthogonal latin square

Dependent variable	MCC^2	F	N	$K-N$
Forced fit				
TOGW	0.951	22	15	17
TMDOD	0.975	44	15	17
PSUBS	0.995	250	15	17
T/W	0.998	754	15	17
TOFL	0.995	243	15	17
Backward step				
TOGW	0.951	28	13	19
TMDOD	0.975	64	12	20
PSUBS	0.995	283	14	18
T/W	[a]	...	...	...
TOFL	0.995	275	14	18
Forward step				
TOGW	0.908	53	5	27
TMDOD	0.969	79	9	23
PSUBS	0.995	425	10	22
T/W	0.998	1403	8	24
TOFL	0.993	286	11	21

[a] Same as forced fit (no regression steps to delete terms).

Table 2 Comparison of TOGW optima for different combinations of experimental design and regression mode

Experimental design/ regression mode	TOGW value	% Error
Faired from data	316600	...
48SLS/forced fit	310969	8.3
48SLS/backstep	310920	8.4
48SLS/forward step	312980	5.3
28OLS/forced fit	311451	7.6
28OLS/backstep	311843	6.9
28OLS/forward step	317491	1.3
52O/forced fit	315921	1.0
52OLS/backstep	315923	1.0
52OLS/forward step	315960	1.0

Table 3 Coefficients of linear correlation for the test case independent variables (48-point random latin square)

	BPR	CPR	SCA	W/S
BPR	1.0	-0.35[a]	-0.05	0.38[a]
CPR		1.0	-0.02	-0.13
SCA			1.0	0.03
W/S				1.0

[a]Significant cross-correlations (95% confidence interval for zero correlation with 48 data points is $|V_{IJ}| \leq 0.28$).

Table 4 Optimization cost in terms of penalty function minimizations and total equivalent function evaluations

Experimental design/ regression mode	Equivalent function evaluations	CDC 6600 accumulated CP time (sec)	Penalty function steps
48SLS/forced fit	255	0.18	2
48SLS/backstep	335	0.23	2
48SLS/forward step	315	0.22	2
28OLS/forced fit	615	0.46	2
28OLS/backstep	625	0.46	2
28OLS/forward step	1795	1.10	3
52OLS/forced fit	440	0.35	2
52OLS/backstep	435	0.34	2
52OLS/forward step	565	0.42	2

These were acceptable for the nine combinations of data selection and regression mode compared here. As mentioned earlier, the regression can be done in three different modes, and these were tried with each of the data selections which were generated by the OLS and SLS methods. The F values shown easily exceed critical table values for 95% confidence and N and $K-N$ degrees of freedom. For this case, the maximum value which N can take is 15, if all terms are in the quadratic polynomial generated, and K is 28, 52, or 48, depending on the data selection used.

Table 2 shows the optimal TOGW value obtained by optimization on the surface fit functions generated by each combination of data selection and regression mode. These are compared with the value faired by hand from the 256-point factorial matrix. It is interesting to note that the OLS selection containing 28 data points yields as good a result as the 52-point selection when forward-step regression mode is used. Some biasing due to repeated data (there were actually 64 data points in the 52 point selection, but 12 were repeats) to obtain usable orthogonal squares may have affected this somewhat, but subsequent analysis has shown that this weighting did not seriously affect the results. Note (Table 3) that the random data selection yields significant cross-correlations between the design variables, and that the optimal TOGW values from the 48-point SLS surface fits (Table 2) are not even as close to the hand-faired optimum as are the 28-point OLS optimal values. Closeness is measured by showing per cent deviation from the hand-faired optimum as a per cent of the total range of TOGW values in the data.

Finally, note that forward-step regression mode appears to be the best way of generating coefficients. For the 6-variable test case, which is shown next, forward-step mode was used almost exclusively, with backward-step mode as a backup for difficult response functions.

It was mentioned that the efficiency of the optimization program is crucial in utilizing the surface fit methods to best advantage. A list of computer run times (CP seconds on the CDC6600) are given in Table 4 for the nine different optimizations which were performed from a single starting point for the 4 variable test case. Equivalent function evaluations (e.f.e.'s) are also shown. To compare the cost of a gradient method with nongradient methods for minimization, the function evaluations called during the gradient-method search are multiplied by $n+1$ (n is the number of design variables) to adjust for computation of the n-dimensional gradient vector. Since $n=4$ in this case, the actual number of times the penalty function was called in each optimization can be found by dividing the table value for e.f.e. by Eq. (5). In this case, the comparison using e.f.e.'s is biased against the gradient method, since gradient calculations are relatively inexpensive for quadratic polynomials. CDC 6600 computer time is a better indicator.

As a final comparison, all 265 data points (the 256 factorial points plus the nine others mentioned earlier) were used to obtain surface fit response functions using the regression program in all three modes. An optimum for each of the 3 sets of approximations so obtained was calculated by the optimization program and compared with the hand-faired optimum in Table 2. Regardless of regression mode, the optima were all nearly identical to the hand-faired optimum.

Recently, a number of cases using the methodology of this paper were used in an evaluation of the effects of airframe variables. For brevity, one of the optima located by the optimization program using surface fit functions is shown in Table 5 alongside simulator values for the response functions at the calculated optimum design variable values. The total evaluation was successful, and the table illustrates the closeness of actual response function values to surface fit response values at an optimum located by the optimization program. In this case, which involved 6 independently varying airframe design parameters, locating an optimum by hand examination of simulator runs on all the factorial matrix data would have been prohibitive.

The airframe variables used were body length (DELBDY), aspect ratio (AR), thickness-to-cord ratio (T/C), wing loading (W/S), thrust-to-weight ratio (T/W), and high lift capability (CLTCH). The response functions in addition to TOGW are listed below with their constraints in the optimization:

Table 5 Optimal surface fit and actual response function values—6-variable test case

Independent variables	DELBDY 6.7	AR 9.8	T/C 0.07[a]	W/S 182.0	T/W 0.454	CLTCH 2.43
Optimal response values:	ACLTMDOD	NPEN	SSDALT	FLTIME	RNG801	
Surface fit	0.825	3.0	50002.0[a]	2.5	8397.0[a]	
Simulation	0.832	2.99	50474.0	2.5	8393.0	
Optimal response values:	TODIST	LDGVEL	RFLTMDOD	RNG802	TOGW	
Surface fit	5505.0	125.0[a]	1.20	2300.0[a]	261425.0	
Simulation	5598.0	125.5	1.243	2275.0	260217.0	

[a]Requirement-constrained values.

acceleration thrust margin	ACLTMDOD	≥ 0.05
penetration max load factor	NPEN	≥ 2.7
dash altitude (supersonic)	SSDALT	≥ 50,000 ft
flush time	FLTIME	≤ 2.6
total range (subsonic)	RNG801	≥ 8350 naut miles
takeoff distance	TODIST	≤ 6000 ft
landing approach speed	LDGVEL	≤ 125 knots
refuel thrust margin	RFLTMDOD	≥ 0.18
total range (supersonic)	RNG802	≥ 2300 naut miles

The box limits on the design variables were:

−20.0	≤	DELBDY	≤	24.0
5.0	≤	AR	≤	11.5
0.07	≤	T/C	≤	0.098
163.0	≤	W/S	≤	217.0
0.333	≤	T/W	≤	0.587
1.0	≤	CLTCH	≤	3.25

The airframe was a supersonic bomber, and the response functions were from both subsonic and supersonic mission runs on the simulator for different combinations of airframe designs using a fixed engine. A total of 68 data points were used, and the regression statistics (mostly forward-step) were good for the surface fits obtained. In this case, $2n^2$ is $2\cdot(6)^2=72$, and this was the goal for data generation by the OLS method. However, some of the airframes would not complete their missions on the simulator properly, and so a smaller data selection was used to complete the evaluation within time constraints.

Computer time for each optimization in six design variables is usually about 1 sec of CP time. There were four response constraints which were active at the optimal solution which is shown here, and one box constraint (these are marked "*a*" in Table 5). An active constraint is one which is actually constraining the optimum away from an unconstrained minimum for the cost response, TOGW.

Conclusions

The test cases presented here should illustrate the conclusion that this methodology is a great improvement over cut-and-try methods for screening analyses. Optimal designs can be located very economically by surface fit and optimization working together. Not only can computer and engineering resources be conserved, but heretofore-impossible analyses can be performed. Important factors affecting airplane design can be isolated in preliminary screening analyses in a comprehensive parametric study.

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